

Mathematics II

029

25/07/2023

08.30 AM -11.30 AM



ADVANCED LEVEL NATIONAL EXAMINATIONS, 2022-2023

SUBJECT: MATHEMATICS II

COMBINATIONS:

- MATHEMATICS-CHEMISTRY-BIOLOGY (**MCB**)
- MATHEMATICS -COMPUTER SCIENCE-ECONOMICS (**MCE**)
- MATHEMATICS-ECONOMICS-GEOGRAPHY (**MEG**)
- MATHEMATICS -PHYSICS-COMPUTER SCIENCE (**MPC**)
- MATHEMATICS-PHYSICS-GEOGRAPHY (**MPG**)
- PHYSICS-CHEMISTRY-MATHEMATICS (**PCM**)

DURATION: 3 HOURS

INSTRUCTIONS:

- 1) Write your names and index number on the answer booklet as written on your registration form, and **DO NOT** write your names and index number on additional answer sheets if provided.
- 2) Do not open this question paper until you are told to do so.
- 3) This paper consists of **two** sections: **A** and **B**.
Section A: Attempt **all** questions. **(55 marks)**
Section B: Attempt **only three** questions. **(45 marks)**
- 4) **Geometrical instruments and silent non-programmable calculators may be used.**
- 5) Use only a **blue** or **black** pen.

SECTION A: ATTEMPT ALL QUESTIONS (55 marks)

- 1) State the cosine and sine laws which are used to solve practical problems involving triangles and angles. **(4 marks)**
- 2) In a geometric progression, insert 4 geometric terms that are between 2 and 6250. **(4 marks)**
- 3) Solve $e^{x-1} - 18e^{1-x} - 3 = 0$ **(4 marks)**
- 4) Find the differential equation of all straight lines passing through the origin. **(2 marks)**
- 5) Find the vector, parametric and symmetric equations of the line (l) passing through the point $A(3, -2, 4)$ with direction vector $\vec{u} = (2, 3, 5)$. **(4 marks)**
- 6) Find square roots of the complex number $3+4i$ **(3 marks)**
- 7) From the top of a cliff, 100 m above sea level, the angle of depression to a ship sailing past is 17 degrees. How far is the ship from the base of the cliff to the nearest meter? **(3 marks)**
- 8) Use Gauss – Jordan method of elimination to solve:

$$\begin{cases} -3x - 2y + 4z = 9 \\ 3y - 2z = 5 \\ 4x - 3y + 2z = 7 \end{cases}$$

(4 marks)

- 9) Write the 3 terms of the Maclaurin expansion of $f(x) = \ln(1 + e^x)$ **(3 marks)**
- 10) Let \mathbf{A} be a 2×2 matrix with $tr(\mathbf{A}) = 6$ and $det(\mathbf{A}) = 5$. Find the eigenvalues of \mathbf{A} . **(3 marks)**
- 11) Let $t: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear transformation such that $t(1,0,0) = (2,4,-1)$, $t(0,1,0) = (1,3,-2)$ and $t(0,0,1) = (1,-2,2)$. Find $t(0,3,-1)$. **(3 marks)**
- 12) Verify if $T = E^2 \rightarrow E^2$ defined by $T((x_1, x_2)) = (x_1 + x_2, x_1 - x_2 + 1)$ is linear or not linear. **(3 marks)**
- 13) Suppose that you are observing the behavior of cell duplication in a laboratory. If, in one of the experiments, you started with 1,000,000 cells and the cell population decreased by ten percent every minute.
- a) Write an equation with base (0.9) to determine the number of cells after t minutes. **(1 mark)**
- b) Determine how long it would take the population to reach a size of 10 cells. **(3 marks)**

$$y = 2x + 1$$

14) Given that: $y = 3x + 1$

$$x = 4$$

a) Find the coordinate points of intersection of the lines. **(3 marks)**

b) Sketch a graph of the given lines in the same two dimensions. **(2 marks)**

c) Find the area of the region found in (b). **(2 marks)**

15) a) Find the constant **c** such that the function :

$$f(x) = \begin{cases} cx^2 & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

is a density function **(2 marks)**

b) Compute $P(1 < x < 2)$ **(2 marks)**

SECTION B: ATTEMPT THREE QUESTIONS ONLY (45 marks)

16) Give that $f(x) = \frac{1}{2}x^2e^{x+1}$

a) Find the domain of $f(x)$, **(1 marks)**

b) Find relative asymptotes (if any), **(4 marks)**

c) Study the first and second derivative with variation table, **(8 marks)**

d) Sketch the curve of $f(x)$. **(2 marks)**

17) Given the differential equation

$$\frac{d^2y}{dx^2} - 2k \frac{dy}{dx} + k^2y = 12xe^{kx}, k > 0$$

a) Find a general solution of differential equation given that $y = Px^3e^{kx}$ where **P** is a constant and part of the solution. **(11 marks)**

b) Given further that $y = 1, \frac{dy}{dx} = 0$ at $x = 0$ show that

$$y = e^{kx}(2x^3 - kx + 1) \quad \textbf{(4 marks)}$$

18) The table below shows the marks scored by 10 students in Biology and Chemistry test.

Biology (x)	8	7	6	9	8	9	7	8	5	6
Chemistry (y)	7	8	7	9	8	8	7	9	7	5

- a) Find Mean, **(5 marks)**
- b) Find Variance, **(1 mark)**
- c) Find Standard deviation of x and y **(1 mark)**
- d) Find covariance of x and y **(1 mark)**
- e) Find correlation coefficient **r** and interpret it **(2 marks)**
- f) Find an equation of a line that best fits in the form of $y = a + bx$ **(3 marks)**
- g) If a student scored 7.5 in Biology, predict his/her score in Chemistry. **(2 marks)**

19) For the conic defined by $9x^2 - 16y^2 - 18x - 64y - 199 = 0$

- a) Write the given conic in a standard form. **(2 marks)**
- b) Name the conic represented in (a). **(1 mark)**
- c) Find the:
 - i) Coordinates of the centre. **(2 marks)**
 - ii) Vertices. **(2 marks)**
 - iii) Foci. **(4 marks)**
 - iv) Equations of the asymptotes then sketch the graph. **(4 marks)**

20) It is given that $Z = \frac{1+8i}{1-2i}$

- a) Express Z in the form $x+iy$, where x and y are real numbers. **(4 marks)**
- b) Find the modulus and argument of Z. **(4 marks)**
- c) Show clearly that: $\arctan 8 + \arctan 2 + \arctan \frac{2}{3} = \pi$ **(7 marks)**

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